4.10 Predicting the Number of Roots of Quadratics

Recall: For a quadratic relation
roots = zeroes = x-intercept = solutions

Given vertex form, look at:
- the location of the vertex (above/below x-axis?)
- the direction of opening (up/down?)

Ex.1 $y = 3(x - 5)^2 - 1$

V(5 , -1)

The vertex lies **above/below?** the x - axis.

The parabola opens **up/down?**

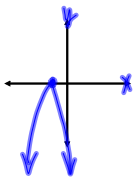
zero(es): 2

Ex.2 $y = -2(x + 1)^2$

V(-1 , 0)

The vertex lies on the x-axis

zero(es): 1



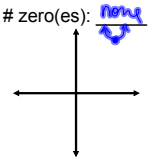
Ex.3 $y = 3(x - 5)^2 + 6$

V(5 , 6).

above/below? the x-axis

opens **up/down?**

zero(es): none



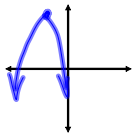
Ex.4 $y = -0.5(x + 3)^2 + 11$

V(-3 , 11).

above/below? the x-axis

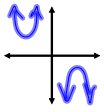
opens **up/down?**

zero(es): 2



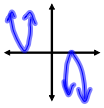
In general to identify the zeros from vertex form:

There will be 0 zeroes if the vertex is above the x-axis and the parabola opens up

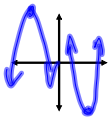


OR if the vertex is below the x-axis and the parabola opens down

There will be 1 zero if the vertex is on the x-axis



There will be 2 zeroes if the vertex is above the x-axis and the parabola opens down



OR if the vertex is below the x-axis and the parabola opens up

What if the quadratic is in standard form? $y = ax^2 + bx + c$

1. Factor and find the number of roots directly.

2. Complete the square (vertex form) and deduce the number of roots by visualizing the graph.

3. Use the quadratic formula if:
   - it cannot be factored
   - the numbers are too difficult to work with

$b^2 - 4ac$

Ex.1 Use the quadratic formula to determine the zeroes.

(a) $-2x^2 - 4x - 2 = 0$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(-2)}}{2(-2)}$

$x = \frac{4 \pm \sqrt{0}}{-4}$

$x = -1$

root(s): 1

root(s): -1

(b) $y = 3x^2 - 30x + 74$

$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(3)(74)}}{2(3)}$

$x = \frac{30 \pm \sqrt{12}}{6}$

$x_1 \doteq 5.58$ $x_2 \doteq 4.42$

root(s): 2

root(s):

(c) $y = 3x^2 - 30x + 81$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(3)(81)}}{2(3)}$

$x = \frac{30 \pm \sqrt{-72}}{6}$

Can't square root a negative number.

root(s): 0

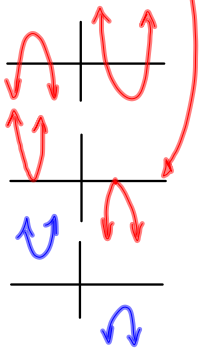
root(s): none / no solutions

$D = b^2 - 4ac$ is called the discriminant.

It tells you how many zeros the quadratic has.

$y = a(x-h)^2$

D is positive $b^2 - 4ac > 0$	two real roots
D is zero $b^2 - 4ac = 0$	one real root (double root)
D is negative $b^2 - 4ac < 0$	no real roots



Ex.2 Find how many zeros each of the following quadratic relations has using the discriminant.

(a) $y = x^2 - 6x + 7$ $D = b^2 - 4ac$ $= (-6)^2 - 4(1)(7)$ $= 8$

∴ there are 2 solutions or 2 distinct roots

(b) $y = 2x^2 - 5x + 9$ $D = -47$ ∴ no roots

(c) $y = x^2 + 3x - 11$ $D = 53$ ∴ 2 distinct roots

(d) $y = 9x^2 - 24x + 16$ $D = 0$ ∴ 1 root

Assigned Work: p.350 # 2, 3, 4, 5, 7, 9, 10, 12